Don’t Panic

- Freakout on Piazza on NY / WV (4 districts = .1 points / 40)
- HW2 released in next few days
- Office hours huge success
Today

- Conditional probability problems
  - Important for classification
  - You’ll be estimating them in HW2
A die is rolled twice

what is the probability that the sum of the faces is greater than 7, given that

- the first outcome was 4?
- the first outcome was greater than 4?
- the first outcome was a 1?
- the first outcome was less than 5?
Dice

- \( p(X_1 + X_2 > 7 \mid X_1 = 4) = \)
- \( p(X_1 + X_2 > 7 \mid X_1 > 4) = \)
- \( p(X_1 + X_2 > 7 \mid X_1 = 1) = \)
- \( p(X_1 + X_2 > 7 \mid X_1 < 5) = \)
Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \phantom{=}$
- $p(X_1 + X_2 > 7 | X_1 > 4) = \phantom{=}$
- $p(X_1 + X_2 > 7 | X_1 = 1) = \phantom{=}$
- $p(X_1 + X_2 > 7 | X_1 < 5) = \phantom{=}
Dice

- \( p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \frac{3}{36} = \frac{1}{12} \)
- \( p(X_1 + X_2 > 7 | X_1 > 4) = \frac{3}{6} = \frac{1}{2} \)
- \( p(X_1 + X_2 > 7 | X_1 = 1) = \frac{5}{12} \)
- \( p(X_1 + X_2 > 7 | X_1 < 5) = \frac{11}{18} \)
Dice

- $p(X_1 + X_2 > 7 \mid X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 \mid X_1 > 4) = \frac{p(X_1 + X_2 > 7 \land X_1 > 4)}{p(X_1 > 4)} = \frac{1}{6}$
- $p(X_1 + X_2 > 7 \mid X_1 = 1) = \frac{p(X_1 + X_2 > 7 \land X_1 = 1)}{p(X_1 = 1)} = \frac{1}{3}$
- $p(X_1 + X_2 > 7 \mid X_1 < 5) = \frac{p(X_1 + X_2 > 7 \land X_1 < 5)}{p(X_1 < 5)} = \frac{1}{9}$
Dice

- \( p(X_1 + X_2 > 7 \mid X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2} \)
- \( p(X_1 + X_2 > 7 \mid X_1 > 4) = \frac{p(X_1 + X_2 > 7 \land X_1 > 4)}{p(X_1 > 4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4} \)
- \( p(X_1 + X_2 > 7 \mid X_1 = 1) = \)
- \( p(X_1 + X_2 > 7 \mid X_1 < 5) = \)
Dice

- $p(X_1 + X_2 > 7 \mid X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 \mid X_1 > 4) = \frac{p(X_1 + X_2 > 7 \land X_1 > 4)}{p(X_1 > 4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 \mid X_1 = 1) = \frac{p(X_1 + X_2 > 7 \land X_1 = 1)}{p(X_1 = 1)} = \frac{9}{36}$
- $p(X_1 + X_2 > 7 \mid X_1 < 5) =$
### Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 | X_1 > 4) = \frac{p(X_1 + X_2 > 7 \land X_1 > 4)}{p(X_1 > 4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1 + X_2 > 7 \land X_1 = 1)}{p(X_1 = 1)} = \frac{0}{1/6} = 0$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$
Dice

- \( p(X_1 + X_2 > 7 \mid X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2} \)

- \( p(X_1 + X_2 > 7 \mid X_1 > 4) = \frac{p(X_1 + X_2 > 7 \land X_1 > 4)}{p(X_1 > 4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4} \)

- \( p(X_1 + X_2 > 7 \mid X_1 = 1) = \frac{p(X_1 + X_2 > 7 \land X_1 = 1)}{p(X_1 = 1)} = \frac{0}{1/6} = 0 \)

- \( p(X_1 + X_2 > 7 \mid X_1 < 5) = \frac{p(X_1 + X_2 > 7 \land X_1 < 5)}{p(X_1 < 5)} = \)
Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1 + X_2 > 7 \land X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 | X_1 > 4) = \frac{p(X_1 + X_2 > 7 \land X_1 > 4)}{p(X_1 > 4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1 + X_2 > 7 \land X_1 = 1)}{p(X_1 = 1)} = \frac{0}{1/6} = 0$
- $p(X_1 + X_2 > 7 | X_1 < 5) = \frac{p(X_1 + X_2 > 7 \land X_1 < 5)}{p(X_1 < 5)} = \frac{6/36}{2/3} = \frac{18}{64} = \frac{1}{4}$
What is the probability a family of two children has two boys
- given that it has at least one boy?
- given that the first child is a boy?
Children

- \( P(X_1 = T, X_2 = T \mid X_1 = T \lor X_2 = T) = \)
- \( P(X_1 = T, X_2 = T \mid X_1 = T) = \)
Children

- \[ P(X_1 = T, X_2 = T \mid X_1 = T \vee X_2 = T) = \frac{P(X_1 = T, X_2 = T)}{P(X_1 = T \vee X_2 = T)} = \]
- \[ P(X_1 = T, X_2 = T \mid X_1 = T) = \]
Children

- \( P(X_1 = T, X_2 = T | X_1 = T \lor X_2 = T) = \frac{P(X_1 = T, X_2 = T)}{P(X_1 = T \lor X_2 = T)} = \frac{1/4}{3/4} = \frac{1}{3} \)
- \( P(X_1 = T, X_2 = T | X_1 = T) = \)
Children

- \( P(X_1 = T, X_2 = T | X_1 = T \lor X_2 = T) = \frac{P(X_1 = T, X_2 = T)}{P(X_1 = T \lor X_2 = T)} = \frac{1/4}{3/4} = \frac{1}{3} \)

- \( P(X_1 = T, X_2 = T | X_1 = T) = \frac{P(X_1 = T, X_2 = T)}{P(X_1 = T)} = \)
Children

- \( P(X_1 = T, X_2 = T | X_1 = T \lor X_2 = T) = \frac{P(X_1 = T, X_2 = T)}{P(X_1 = T \lor X_2 = T)} = \frac{1/4}{3/4} = \frac{1}{3} \)

- \( P(X_1 = T, X_2 = T | X_1 = T) = \frac{P(X_1 = T, X_2 = T)}{P(X_1 = T)} = \frac{1/4}{1/2} = \frac{1}{2} \)
Conditional Probabilities

One coin in a collection of 65 has two heads. The rest are fair. If a coin, chosen at random from the lot and then tossed, turns up heads 6 times in a row, what is the probability that it is the two-headed coin?
Conditional Probabilities

- Let $C$ be the coin chose ($T$ for fake)
- Let $H$ be the number of heads out of six

$$P(C = T | H = 6) =$$ (1)
Conditional Probabilities

- Let $C$ be the coin chose ($\top$ for fake)
- Let $H$ be the number of heads out of six

\[
P(C = \top | H = 6) = \frac{P(C = \top \wedge H = 6)}{P(H = 6)} =
\]
Conditional Probabilities

- Let $C$ be the coin chose ($\top$ for fake)
- Let $H$ be the number of heads out of six

\[
P(C = \top \mid H = 6) = \frac{P(C = \top \land H = 6)}{P(H = 6)} = \frac{1/65}{1/65 + \frac{64}{65} \cdot \frac{1}{2^6}} = (1)
\]
Conditional Probabilities

- Let $C$ be the coin chose ($\top$ for fake)
- Let $H$ be the number of heads out of six

\[
P(C = \top | H = 6) = \frac{P(C = \top \land H = 6)}{P(H = 6)} = \frac{1/65}{1/65 + \frac{64}{65} \cdot \frac{1}{2^6}} = \frac{1/65}{2/65} = \frac{1}{2} \quad (1)
\]
Bayes Rule

There’s a test for Boogie Woogie Fever (BWF). The probability of getting a positive test result given that you have BWF is 0.8, and the probability of getting a positive result given that you do not have BWF is 0.01. The overall incidence of BWF is 0.01.

1. What is the marginal probability of getting a positive test result?
2. What is the probability of having BWF given that you got a positive test result?
Bayes Rule

Let $D$ be the disease, $T$ be the test

- $P(T = T) =$
- $P(D = T | T = T) =$
Bayes Rule

Let $D$ be the disease, $T$ be the test

- $P(T = T) = \sum_{x=T,\bot} P(T = T, D = x) =$
- $P(D = T | T = T) =$

\[ P(T = T) = \sum_{x=T,\bot} P(T = T, D = x) = \]
\[ P(D = T | T = T) = \]
Bayes Rule

Let $D$ be the disease, $T$ be the test

- $P(T = T) = \sum_{x=T,\perp} P(T = T, D = x) = 0.01 \cdot 0.8 + 0.99 \cdot 0.01 =$
- $P(D = T \mid T = T) =$
Bayes Rule

Let $D$ be the disease, $T$ be the test

- $P(T = T) = \sum_{x \in \{T, \bot\}} P(T = T, D = x) = 0.01 \cdot 0.8 + 0.99 \cdot 0.01 = 0.02$
- $P(D = T | T = T) =$

"Data Science: Jordan Boyd-Graber | UMD"
Bayes Rule

Let $D$ be the disease, $T$ be the test

- $P(T = \top) = \sum_{x=\top,\bot} P(T = \top, D = x) = 0.01 \cdot 0.8 + 0.99 \cdot 0.01 = 0.02$
- $P(D = \top | T = \top) = \frac{P(T = \top | D = \top) P(D = \top)}{P(T = \top)} = \frac{P(T = \top | D = \top) P(D = \top)}{P(T = \top)}$
Bayes Rule

Let $D$ be the disease, $T$ be the test

- $P(T = \top) = \sum_{x=\top,\bot} P(T = \top, D = x) = 0.01 \cdot 0.8 + 0.99 \cdot 0.01 = 0.02$
- $P(D = \top | T = \top) = \frac{P(T = \top | D = \top)P(D = \top)}{P(T = \top)} = \frac{0.8 \cdot 0.01}{0.02} = \ldots$
Bayes Rule

Let $D$ be the disease, $T$ be the test

- $P(T = T) = \sum_{x=T,\perp} P(T = T, D = x) = 0.01 \cdot 0.8 + 0.99 \cdot 0.01 = 0.02$
- $P(D = T | T = T) = \frac{P(T = T | D = T)P(D = T)}{P(T = T)} = \frac{0.8 \cdot 0.01}{0.02} = 0.4$