Online Latent Dirichlet Allocation with Infinite Vocabulary

Ke Zhai
Department of Computer Science, University of Maryland, College Park, MD USA

Jordan Boyd-Graber
iSchool and UMIACS, University of Maryland, College Park, MD USA

Abstract

Topic models based on latent Dirichlet allocation (LDA) assume a predefined vocabulary. This is reasonable in batch settings but not reasonable for streaming and online settings. To address this lacuna, we extend LDA by drawing topics from a Dirichlet process whose base distribution is a distribution over all strings rather than from a finite Dirichlet. We develop inference using online variational inference and—to only consider a finite number of words for each topic—propose heuristics to dynamically order, expand, and contract the set of words we consider in our vocabulary. We show our model can successfully incorporate new words and that it performs better than topic models with finite vocabularies in evaluations of topic quality and classification performance.

1. Introduction

Latent Dirichlet allocation (LDA) is a probabilistic approach for exploring topics in document collections (Blei et al., 2003). Topic models offer a formalism for exposing a collection’s themes and have been used to aid information retrieval (Wei & Croft, 2006), understand academic literature (Dietz et al., 2007), and discover political perspectives (Paul & Girju, 2010).

As hackneyed as the term “big data” has become, researchers and industry alike require algorithms that are scalable and efficient. Topic modeling is no different. A common scalability strategy is converting batch algorithms into streaming algorithms that only make one pass over the data. In topic modeling, Hoffman et al. (2010) extended LDA to online settings.

However, this and later online topic models (Wang et al., 2011; Mimno et al., 2012) make the same limiting assumption. The namesake topics, distributions over words that evince thematic coherence, are always modeled as a multinomial drawn from a finite Dirichlet distribution. This assumption precludes additional words being added over time. Particularly for streaming algorithms, this is neither reasonable nor appealing. There are many reasons immutable vocabularies do not make sense: words are invented (“crowdsourcing”), words cross languages (“Gangnam”), or words common in one context become prominent elsewhere (“vuvuzelas” moving from music to sports in the 2010 World Cup). To be flexible, topic models must be able to capture the addition, invention, and increased prominence of new terms.

Allowing models to expand topics to include additional words requires changing the underlying statistical formalism. Instead of assuming that topics come from a finite Dirichlet distribution, we assume that it comes from a Dirichlet process (Ferguson, 1973) with a base distribution over all possible words, of which there are an infinite number. Bayesian nonparametric tools like the Dirichlet process allow us to reason about distributions over infinite supports. We review both topic models and Bayesian nonparametrics in Section 2.

In Section 3, we present the infinite vocabulary topic model, which uses Bayesian nonparametrics to go beyond fixed vocabularies.

In Section 4, we derive approximate inference for our model. Since emerging vocabulary are most important in non-batch settings, in Section 5, we extend inference to streaming settings. We compare the coherence and effectiveness of our infinite vocabulary topic model against models with fixed vocabulary in Section 6.

Figure 1 shows a topic evolving during inference. The algorithm processes documents in sets we call minibatches; after each minibatch, online variational inference updates our model’s parameters. This shows that out of vocabulary words can enter topics and eventually become high probability words.
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New-Words 2: hiram<br>corpus<br>moskowitz<br>moskowitz
New-Words 3: crohn<br>crim<br>crim<br>crim<br>crohn
New-Words 5: captain<br>captain<br>captain<br>captain<br>captain<br>captain<br>captain<br>captain
New-Words 8: comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip<br>comicstrip
New-Words 10:uo<br>uo<br>uo<br>uo<br>uo
New-Words 16: hawk<br>hawk<br>hawk<br>hawk<br>hawk
New-Words 17: lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai<br>lai
New-Words 30: izzo<br>izzo<br>izzo
New-Words 83: gwen<br>gwen<br>gwen

Figure 1. The evolution of a single “comic book” topic from the 20 newsgroups corpus. Each column is a ranked list of word probabilities after processing a minibatch (numbers preceding words are the exact rank). The box below the topics contains words introduced in a minibatch. For example, “hulk” first appeared in minibatch 10, was ranked at 9659 after minibatch 17, and became the second most important word by the final minibatch. Colors help show words’ trajectories.

2. Background

Latent Dirichlet allocation (Blei et al., 2003) assumes a simple generative process. The $K$ topics, drawn from a symmetric Dirichlet distribution, $\beta_k \sim \text{Dir}(\eta)$, $k = \{1, \ldots, K\}$ generate a corpus of observed words:

1. for each document $d$ in a corpus $D$ do
2. Choose a distribution $\theta_d$ over topics from a Dirichlet distribution $\theta_d \sim \text{Dir}(\alpha^D)$.
3. for each of the $n = 1, \ldots, N_d$ word indexes do
4. Choose a topic $z_n$ from the document’s distribution over topics $z_n \sim \text{Mult}(\theta_d)$.
5. Choose a word $w_n$ from the appropriate topic’s distribution over words $p(w_n | z_n, \beta_{z_n})$.

Implicit in this model is a finite number of words in the vocabulary because the support of the Dirichlet distribution $\text{Dir}(\eta)$ is fixed. Moreover, it fixes a priori which words we can observe, a patently false assumption (Algeo, 1980).

2.1. Bayesian Nonparametrics

Bayesian nonparametrics is an appealing solution; it models arbitrary distributions with an unbounded and possibly countably infinite support. While Bayesian nonparametrics is a broad field, we focus on the Dirichlet process (DP, Ferguson 1973).

The Dirichlet process is a two-parameter distribution with scale parameter $\alpha^D$ and base distribution $G_0$. A draw $G$ from DP($\alpha^D, G_0$) is modeled as

$$ b_1, \ldots, b_n, \ldots \sim \text{Beta}(1, \alpha^D), \quad \rho_1, \ldots, \rho_n, \ldots \sim G_0.$$

Individual draws from a Beta distribution are the foundation for the stick-breaking construction of the DP ( Sethuraman, 1994). Each break point $b_i$ models how much probability mass remains. These break points combine to form an infinite multinomial,

$$ \beta_i \equiv b_i \prod_{j=1}^{i-1} (1 - b_j), \quad G \equiv \sum_i \beta_i \delta_{\rho_i}, \quad (1)$$
where the weights $\beta_i$ give the probability of selecting any particular atom $\rho_i$ from the base distribution.

The model we develop in Section 3 uses a base distribution over all possible words, and each topic is a draw from the Dirichlet process. This approach is inspired by unsupervised models that induce parts-of-speech.

### 2.2. N-gram Models in Latent Variable Models

A strength of the probabilistic formalism is the ability to embed specialized models inside more general models. The problem of part-of-speech (POS) induction (Goldwater & Griffiths, 2007) uses morphological regularity within part of speech classes (e.g., verbs in English often end with “ed”) to learn a character n-gram model for parts of speech (Clark, 2003). This has been combined within the latent variable HMM via a Chinese restaurant process (Blunsom & Cohn, 2011).

We also view latent clusters of words (topics) as a non-parametric distribution with a character n-gram base distribution, but to better support streaming data sets, we use online variational inference; previous approaches used Monte Carlo methods (Neal, 1993). Variational approaches are easier to distribute (Zhai et al., 2012) and amenable to online updates (Hoffman et al., 2010).

Within the topic modeling community, there are different approaches to deal with changing word use. Dynamic topic models (Blei & Lafferty, 2006) discover evolving topics by viewing word distributions as $n$-dimensional points undergoing Brownian motion. These models reveal compelling topical evolution; e.g., physics moving from studies of the ether to relativity to quantum mechanics. However, the models assume fixed vocabularies; we show that our infinite vocabulary model discovers more coherent topics (Section 6.2).

An elegant solution for large vocabularies is the “hashing trick” (Weinberger et al., 2009), which maps strings into a restricted set of integers via a hash function. These integers become the topic model’s vocabulary. While elegant, words are no longer identifiable. However, our infinite vocabulary topic model retains identifiability and better models datasets (Section 6.3).

### 3. Infinite Vocabulary Topic Model

Our generative process is identical to LDA’s (Section 2) except that topics are not drawn from a finite Dirichlet. Instead, topics are drawn from a DP with base distribution $G_0$ over all possible words:

1: for each topic $k$ do
2: Draw words $\rho_{ks}, \{s = 1, 2, \ldots\}$ from $G_0$.
3: Draw $b_{kt} \sim \text{Beta}(1, \alpha^2), \{t = 1, 2, \ldots\}$.

4: Set stick weights $\beta_{kt} = b_{kt} \prod_{s \leq t} (1 - b_{ks})$.

The rest is identical to LDA.

#### 3.1. A Distribution over Words

An intuitive choice for $G_0$ is a conventional character language model. However, such a naïve approach is unrealistic and is biased to shorter words; preliminary experiments yielded poor results. Instead, we define $G_0$ as the following distribution over strings:

1: Choose a length $l \sim \text{Mult}(\lambda)$.
2: Generate character $c_1 \sim p(c_1|c_{-1}, \ldots, s_{-1})$.

This is similar to the classic $n$-gram language model, except that the length is first chosen from a multinomial distribution over all lengths. Estimating conditional $n$-gram probabilities is well-studied in natural language processing (Jelinek & Mercer, 1985).

The full expression for the probability of a word $\rho$ consisting of the characters $c_1, c_2, \ldots$ under $G_0$ is

$$G_0(\rho) \equiv p_{\text{NM}}(l = |\rho| | \lambda) \prod_{t=1}^{l}| \rho_{\text{WM}}(c_t|c_{t-n, \ldots, t-1})|$$

where $|\rho|$ is the length of the word. To avoid length bias, we chose the multinomial $\lambda$ that minimizes the average discrepancy between word corpus probabilities $p_C$ and the probability in our word model

$$\lambda \equiv \arg\min_{\lambda} \sum_{\rho} |p_C(\rho) - p_{\text{NM}}(\rho | \lambda)|^2, \text{s.t. } \sum_{\lambda} \lambda = 1.$$ 

The $n$-gram statistics are estimated from an English dictionary which need not be very large, since it is a language model over characters, not words.

#### 4. Variational Approximation

Inference in probabilistic inference uncovers the latent variables that best reconstruct observed data. The quality of this reconstruction is measured by log likelihood. For a corpus of $D$ documents where the $d$-th document contains $N_d$ words, the joint distribution is

$$p(W, \rho, \beta, \theta, z) = \prod_{k=1}^{K} \left( \prod_{t=1}^{n} p(\rho_{ks} | G_0) \cdot p(\beta_{ts} | \alpha^2) \right) \prod_{d=1}^{D} p(\theta_d | \phi) \prod_{t=1}^{n} p(z_{dn} | \theta_d) p(z_{dn} | \omega_{dn} | z_{dn}, \beta_{zn}).$$

Directly optimizing the latent variables $Z \equiv \{\text{corpus-level stick proportions } \beta, \text{ document topic distributions } \theta \text{ and word topic assignments } z\}$ is intractable, so we use variational inference (Blei et al., 2003).

To use variational inference, we select a simpler family of distributions over the latent variables $Z$. We call these distributions $q$. This family of distributions allows us to optimize a lower bound of the likelihood called the evidence lower bound (ELBO) $L$,
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\[ \log p(W) \geq E_q(Z) \left[ \log p(W, Z) \right] - E_q(Z) \left[ q \right] = \mathcal{L}. \] (2)

Maximizing \( \mathcal{L} \) is equivalent to minimizing the Kullback-Leibler (KL) divergence between the true distribution and the variational distribution.

Unlike mean-field approaches (Blei et al., 2003), which assume \( q \) is a fully factorized distribution, we integrate out the word-level topic distribution vector \( \Theta \): \( q(z_d | \eta) \) is a single distribution over \( K^N_d \) possible topic configurations rather than a product of \( N_d \) multinomial distributions over \( K \) topics. Combined with a beta distribution \( q(b_k | \nu^k_1, \nu^k_2) \) for stick break points, the variational distribution \( q \) is

\[ q(Z) \equiv q(\beta, z) = \prod_D q(z_d | \eta) \prod_K q(b_k | \nu^k_1, \nu^k_2). \] (3)

However, we cannot explicitly represent a distribution over all possible strings, so we truncate our variational stick-breaking distribution \( q(b | \nu) \) to a finite set.

### 4.1. Truncation Ordered Set

Variational methods typically cope with infinite dimensionality of nonparametric models by truncating the distribution to a finite subset of all possible atoms that nonparametric distributions consider (Blei & Jordan, 2005; Kurihara et al., 2006; Boyd-Graber & Blei, 2009). This is done by selecting a relatively large truncation index \( T_k \), and then stipulating that the variational distribution uses the rest of the available stick at that index, i.e., \( q(b_{ts} = 1) \equiv 1 \). As a consequence, \( \beta \) is zero in expectation under \( q \) beyond that index.

However, directly applying such a technique is not feasible here, as truncation is not just a search over dimensionality but also over atom strings and their ordering. This is often a problem in nonparametric models, and the truncation that solves the problem matches the underlying probabilistic model: for mixture models, it is the number of components (Blei & Jordan, 2005); for hierarchical topic models, it is a tree (Wang & Blei, 2009); for natural language grammars, it is grammatoms (Cohen et al., 2010). Similarly, our truncation is not just a fixed vocabulary size; it is a truncation ordered set (TOS). The ordering is important because the Dirichlet process is a size-biased distribution; words with lower indices are likely to have a higher probability than words with higher indices.

Each topic has a unique TOS \( T_k \) of limited size that maps every word type \( w \) to an integer \( t \); thus \( t = T_k(w) \) is the index of the atom \( \rho_{kt} \) that corresponds to \( w \). We defer how we choose this mapping until Section 4.3. More pressing is how we compute the two variational distributions of interest. For \( q(z | \eta) \), we use local collapsed MCMC sampling (Mimno et al., 2012) and for \( q(b | \nu) \) we use stochastic variational inference (Hoffman et al., 2010). We describe both in turn.

### 4.2. Stochastic Inference

Recall that the variational distribution \( q(z_d | \eta) \) is a single distribution over the \( N_d \) vectors of length \( K \). While this removes the tight coupling between \( \theta \) and \( z \) that often complicates mean-field variational inference, it is no longer as simple to determine the variational distribution \( q(z_d | \eta) \) that optimizes Eqn. (2). However, Mimno et al. (2012) showed that Gibbs sampling instantiations of \( z_{dn} \) from the distribution conditioned on other topic assignments results in a sparse, efficient empirical estimate of the variational distribution. In our model, the conditional distribution of a topic assignment of a word with TOS index \( t = T_k(w_{dn}) \) is

\[ q(z_{dn} = k | z_{dn}, t = T_k(w_{dn})) \propto \left( \sum_{m = 1}^{N_d} \mathbb{I}_{z_{an} = k + \alpha^k_b} \exp \{ E_q(\omega) \log \beta_k \} \right). \] (4)

We iteratively sample from this conditional distribution to obtain the empirical distribution \( \phi_{dn} \equiv \hat{q}(z_{dn}) \) for latent variable \( z_{dn} \), which is fundamentally different from mean-field approach (Blei et al., 2003).

There are two cases to consider for computing Eqn. (4)—whether a word \( w_{dn} \) is in the TOS for topic \( k \) or not. First, we look up the word’s index \( t = T_k(w_{dn}) \). If this word is in the TOS, i.e., \( t \leq T_k \), the expectations are straightforward (Mimno et al., 2012)

\[ q(z_{dn} = k) \propto \left( \sum_{m = 1}^{N_d} \phi_{dnk} + \alpha^k_b \right) \cdot \exp \{ \Psi(z_{dn}) \} \] (5)

\[ + \sum_{s = 1}^{c_{dn}} \Psi(\nu^k_{ds}) - \sum_{s = 1}^{c_{dn}} \Psi(\nu^k_{ds} + \nu^k_{es}) \]

It is more complicated when a word is not in the TOS. Wang & Blei (2012) proposed a truncation-free stochastic variational approach for DPs. It provides more flexible truncation schemes than split-merge techniques (Wang & Blei, 2009). The algorithm resembles a collapsed Gibbs sampler; it does not represent all components explicitly. For our infinite vocabulary topic model, we do not ignore out of vocabulary (OOV) words; we assign these unseen words probability \( 1 - \sum_{z \leq T_k} \exp \{ E_q(\omega) \log \beta_k \} \). The conditional distribution of an unseen word \( t > T_k \) is then

\[ q(z_{dn} = k) \propto \left( \sum_{m = 1}^{N_d} \phi_{dnk} + \alpha^k_b \right) \] (6)

\[ \cdot \exp \{ \sum_{s = 1}^{c_{dn}} (\Psi(\nu^k_{ds}) - \Psi(\nu^k_{ds} + \nu^k_{es})) \} \].

This is different from finite vocabulary topic models that set vocabulary a priori and ignore OOV words.

### 4.3. Refining the Truncation Ordered Set

In this section, we describe heuristics to update the TOS inspired by MCMC conditional equations, a common practice for updating truncations. One component of a good TOS is that more frequent words should come first in the ordering.
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This is reasonable because the stick-breaking prior induces a size-biased ordering of the clusters. This has previously been used for truncation optimization for Dirichlet process mixtures and admixtures (Kurihara et al., 2007).

Another component of a good TOS is that words consistent with the underlying base distribution should be ranked higher than those not consistent with the base distribution. This intuition is also consistent with the conditional sampling equations for MCMC inference (Müller & Quintana, 2004); the probability of creating a new table with dish ρ is proportional to \( \alpha^\beta G_0(\rho) \) in the Chinese restaurant process.

Thus, to update the TOS, we define the ranking score of word \( t \) in topic \( k \) as

\[
R(\rho_{kt}) = p(\rho_{kt}|G_0) \sum_{d=1}^{D} \sum_{n=1}^{N_d} \phi_{dnk} \delta_{\omega_d=\rho_{kt}},
\]

(7)

sort all words by the scores within that topic, and then use those positions as the new TOS. In Section 5.1, we present online updates for the TOS.

5. Online Inference

Online variational inference seeks to optimize the ELBO \( \mathcal{L} \) according to Eqn. (2) by stochastic gradient optimization. Because gradients estimated from a single observation are noisy, stochastic inference for topic models typically uses “ minibatches” of \( S \) documents out of \( D \) total documents (Hoffman et al., 2010).

An approximation of the natural gradient of \( \mathcal{L} \) with respect to \( \nu \) is the product of the inverse Fisher information and its first derivative (Sato, 2001)

\[
\Delta \nu_{kt}^1 = 1 + \frac{D}{|S|} \sum_{d \in S} \sum_{n=1}^{N_d} \phi_{dnk} \delta_{\omega_d=\rho_{kt}} - \nu_{kt}^1 \quad (8)
\]

\[
\Delta \nu_{kt}^2 = \alpha^\beta + \frac{D}{|S|} \sum_{d \in S} \sum_{n=1}^{N_d} \phi_{dnk} \delta_{\omega_d>\rho_{kt}} - \nu_{kt}^2,
\]

which leads to an update of \( \nu \),

\[
\nu_{kt}^1 = \nu_{kt}^1 + \epsilon \cdot \Delta \nu_{kt}^1, \quad \nu_{kt}^2 = \nu_{kt}^2 + \epsilon \cdot \Delta \nu_{kt}^2 \quad (9)
\]

where \( \epsilon_i = (\tau_0 + i)^{-\kappa} \) defines the step size of the algorithm in minibatch \( i \). The learning rate \( \kappa \) controls how quickly new parameter estimates replace the old; \( \kappa \in (0.5, 1] \) is required for convergence. The learning inertia \( \tau_0 \) prevents premature convergence. We recover the batch setting if \( S = D \) and \( \kappa = 0 \).

5.1. Updating the Truncation Ordered Set

A nonparametric streaming model should allow the vocabulary to dynamically expand as new words appear (e.g., introducing “vuvuzelas” for the 2010 World Cup), and contract as needed to best model the data (e.g., removing “vuvuzelas” after the craze passes). We describe three components of this process, expanding the truncation, refining the ordering of TOS, and contracting the vocabulary.

Determining the TOS Ordering This process depends on the ranking score of a word in topic \( k \) at minibatch \( i \), \( R_{i,k}(\rho) \). Ideally, we would compute \( R \) from all data. However, only a single minibatch is accessible. We have a per-minibatch rank estimate

\[
r_{i,k}(\rho) = p(\rho|G_0) \cdot \frac{|S|}{|S|} \sum_{d \in S} \sum_{n=1}^{N_d} \phi_{dnk} \delta_{\omega_d=\rho}
\]

which we interpolate with our previous ranking

\[
R_{i,k}(\rho) = (1 - \epsilon) \cdot R_{i-1,k}(\rho) + \epsilon \cdot r_{i,k}(\rho).
\]

(10)

We introduce an additional algorithm parameter, the reordering delay \( U \). We found that reordering after every minibatch (\( U = 1 \)) was not effective; we explore the role of reordering delay in Section 6. After \( U \) minibatches have been observed, we reorder the TOS for each topic according to the words’ ranking score \( R \) in Eqn. (10); \( T_k(w) \) becomes the rank position of \( w \) according to the latest \( R_{i,k} \).

Expanding the Vocabulary Each minibatch contains words we have not seen before. When we see them, we must determine their relative rank position in the TOS, their rank scores, and their associated variational parameters. The latter two issues are relevant for online inference because both are computed via interpolations from previous values in Eqn. (10) and (9). For an unseen word \( \omega \), previous values are undefined. Thus, we set \( R_{i-1,k} \) for unobserved words to be 0, \( \nu \) to be 1, and \( T_k(\omega) \) is \( T_k + 1 \) (i.e., increase truncation and append to the TOS).

Contracting the Vocabulary To ensure tractability we must periodically prune the words in the TOS. When we re-order the TOS (after every \( U \) minibatches), we only keep the top \( T \) terms, where \( T \) is a user-defined integer. A word type \( \rho \) will be removed from \( T_k \) if its index \( T_k(\rho) > T \) and its previous information (e.g., rank and variational parameters) is discarded. In a later minibatch, if a previously discarded word reappears, it is treated as a new word.

6. Experimental Evaluation

In this section, we evaluate the performance of our infinite vocabulary topic model (infvoc) on two corpora: \textit{de-news}\textsuperscript{1} and 20 newsgroups\textsuperscript{2}. Both corpora were parsed by the same

\textsuperscript{1}A collection of daily news items between 1996 to 2000 in English. It contains 9,756 documents, 1,175,526 word tokens, and 20,000 distinct word types. Available at homepages.inf.ed.ac.uk/pkoehn/publications/de-news.

\textsuperscript{2}A collection of discussions in 20 different newsgroups. It contains 18,846 documents and 100,000 distinct word types. It
tokenizer and stemmer with a common English stopword list (Bird et al., 2009). First, we examine its sensitivity to both model parameters and online learning rates. Having chosen those parameters, we then compare our model with other topic models with fixed vocabularies.

**Evaluation Metric** Typical evaluation of topic models is based on held-out likelihood or perplexity. However, creating a strictly fair comparison for our model against existing topic model algorithms is difficult, as traditional topic model algorithms must discard words that have not previously been observed. Moreover, held-out likelihood is a flawed proxy for how topic models are used in the real world (Chang et al., 2009). Instead, we use two evaluation metrics: topic coherence and classification accuracy.

**Pointwise mutual information (PMI)**, which correlates with human perceptions of topic coherence, measures how words fit together within a topic. Following Newman et al. (2009), we extract document co-occurrence statistics from Wikipedia and score a topic’s coherence by averaging the pairwise PMI score (w.r.t. Wikipedia co-occurrence) of the topic’s ten highest ranked words. Higher average PMI implies a more coherent topic.

**Classification accuracy** is the accuracy of a classifier learned from the topic distribution of training documents applied to test documents (the topic model sees both sets). A higher accuracy means the unsupervised topic model better captures the underlying structure of the corpus. To better simulate real-world situations, 20-newsgroup’s test/train split is by date (test documents appeared after training documents).

**Comparisons** We evaluate the performance of our model (infvoc) against three other models with fixed vocabularies: online variational Bayes LDA (fixvoc-vb, Hoffman et al. 2010), online hybrid LDA (fixvoc-hybrid, Mimno et al. 2012), and dynamic topic models (dtm, Blei & Lafferty 2006). Including dynamic topic models is not a fair comparison as its inferences requires access to all of the documents in the dataset; unlike the other algorithms, it is not online.

**Vocabulary** For fixed vocabulary models, we must decide on a vocabulary a priori. We consider two different vocabulary methods: use the first minibatch to define a vocabulary (null) or use a comprehensive dictionary³ (dict). We use the same dictionary to train infvoc’s base distribution.

**Experiment Configuration** For all models, we use the same symmetric document Dirichlet prior with \( \alpha_0 = 1/K \), where \( K \) is the number of topics. Online models see exactly the same minibatches. For dtm, which is not an online

Figure 2. PMI score on de-news (Figure 2(a) and 2(b), \( K = 10 \)) and 20 newsgroups (Figure 2(c) and 2(d), \( K = 50 \)) against different settings of DP scale parameter \( \alpha_0 \), truncation level \( T \) and reordering delay \( U \), under learning rate \( \kappa = 0.8 \) and learning inertia \( \tau_0 = 64 \). Our model is more sensitive to \( \alpha_0 \) and less sensitive to \( T \).

Figure 3. PMI score on two datasets with reordering delay \( U = 20 \) against different settings of decay factor \( \kappa \) and \( \tau_0 \). A suitable choice of DP scale parameter \( \alpha_0 \) increases the performance significantly. Learning parameters \( \kappa \) and \( \tau_0 \) jointly define the step decay. Larger step sizes promote better topic evolution.

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³http://sll.org/linguistics/wordlists/english/
algorithm but instead partitions its input into “epochs”, we combine documents in ten consecutive minibatches into an epoch (longer epochs tended to have worse performance; this was the shortest epoch that had reasonable runtime).

For online hybrid approaches (infvoc and fixvoc-hybrid), we collect 10 samples empirically from the variational distribution in E-step with 5 burn-in sweeps. For fixvoc-vb, we run 50 iterations for local parameter updates.

6.1. Sensitivity to Parameters

Figure 2 shows how the PMI score is affected by the DP scale parameter $\alpha^2$, the truncation level $T$, and the reordering delay $U$. The relatively high values of $\alpha^3$ may be surprising to readers used to seeing a DP that instantiates dozens of atoms, but when vocabularies are in tens of thousands, such scale parameters are necessary to support the long tail. Although we did not investigate such approaches, this suggests that more advanced nonparametric distributions (Teh, 2006) or explicitly optimizing $\alpha^3$ may be useful. Relatively large values of $U$ suggest that accurate estimates of the rank order are important for maintaining coherent topics.

While infvoc is sensitive to parameters related to the vocabulary, once suitable values of those parameters are chosen, it is no more sensitive to learning-specific parameters than other online LDA algorithms (Figure 3), and values used for other online topic models also work well here.

Table 1. Classification accuracy based on 50 topic features extracted from 20 newsgroups data. Our model (infvoc) out-performs algorithms with a fixed or hashed vocabulary but not dtm, a batch algorithm that has access to all documents.

6.2. Comparing Algorithms: Coherence

Now that we have some idea of how we should set parameters for infvoc, we compare it against other topic modeling techniques. We used grid search to select parameters for each of the models4 and plotted the topic coherence averaged over all topics in Figure 4.

While infvoc initially holds its own against other models, it does better and better in later minibatches, since it has managed to gain a good estimate of the vocabulary and the topic distributions have stabilized. Most of the gains in topic coherence come from highly specific proper nouns which are missing from vocabularies of the fixed-vocabulary topic models. This advantage holds even against dtm, which uses batch inference.

6.3. Comparing Algorithms: Classification

For the classification comparison, we consider additional topic models. While we need the most probable topic strings for PMI calculations, classification experiments only need a document’s topic vector. Thus, we consider hashed vocabulary schemes. The first, which we call dict-hashing, uses a dictionary for the known words and hashes any other words

- For the de-news dataset, we select (20 newsgroups parameters in parentheses) minibatch size $S \in \{140, 245\}$ ($S \in \{155, 310\}$), DP scale parameter $\alpha^2 \in \{1k, 2k\}$ ($\alpha^3 \in \{3k, 4k, 5k\}$), truncation size $T \in \{3k, 4k\}$ ($T \in \{20k, 30k, 40k\}$), reordering delay $U \in \{10, 20\}$ for fixvoc; and topic chain variable $tvc \in \{0.001, 0.005, 0.01, 0.05\}$ for dtm.
into the same set of integers. The second, full-hash, used in Vowpal Wabbit, hashes all words into a set of $T$ integers.

We train 50 topics for all models on the entire dataset and collect the document level topic distribution for every article. We treat such statistics as features and train a SVM classifier on all training data using Weka (Hall et al., 2009) with default parameters. We then use the classifier to label testing documents with one of the 20 newsgroup labels. A higher accuracy means the model is better capturing the underlying content.

Our model infvoc captures better topic features than online LDA fixvoc (Table 1) under all settings. This suggests that in a streaming setting, infvoc can better categorize documents. However, the batch algorithm dtm, which has access to the entire dataset performs better because it can use later documents to retrospectively improve its understanding of earlier ones. Unlike dtm, infvoc only sees early minibatches once and cannot revise its model when it is tested on later minibatches.

6.4. Qualitative Example

Figure 1 shows the evolution of a topic in 20 newsgroups about comics as new vocabulary words enter from new minibatches. While topics improve over time (e.g., relevant words like “seri(es)”, “issu(e)”, “forc(e)” are ranked higher), interesting words are being added throughout training and become prominent after later minibatches are processed (e.g., “captain”, “comicstrip”, “mutant”). This is not the case for standard online LDA—these words are ignored and the model does not capture such information. In addition, only about 60% of the word types appeared in the SIL English dictionary. Even with a comprehensive English dictionary, online LDA could not capture all the word types in the corpus, especially named entities.

7. Conclusion and Future Work

We proposed an online topic model that, instead of assuming vocabulary is known a priori, adds and sheds words over time. While our model is better able to create coherent topics, it does not outperform dynamic topic models (Blei & Jordan, 2009), our approach is more robust and incorporating the our approach with models of transliteration (Knight & Graehl, 1997) might allow concepts expressed in one language better capture concepts in another, further improving the ability of algorithms to capture the evolving themes and topics in large, streaming datasets.

Acknowledgments

The authors thank Chong Wang, Dave Blei, and Matt Hoffman for answering questions and sharing code. We thank Jimmy Lin and the anonymous reviewers for helpful suggestions. Research supported by NSF grant #1018625. Any opinions, conclusions, or recommendations are the authors’ and not those of the sponsors.

References


Online Latent Dirichlet Allocation with Infinite Vocabulary


